

Bound and scattering states in harmonic waveguides in the vicinity of free space Feshbach resonances

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Abstract. The two-body bound and scattering properties in an one-dimensional harmonic waveguide close to free space magnetic Feshbach resonances are investigated based on the local frame transformation approach within a single partial wave approximation. An energy and magnetic field dependent free space phase shift is adopted in the current theoretical framework. For both s - and p -wave interaction, the least bound state in the waveguide dissociates into the continuum at the resonant magnetic field where the effective one-dimensional scattering length a_{1D} diverges. Consequently, the association of atoms into molecules in the waveguide occurs when the magnetic field is swept adiabatically across the pole of a_{1D} . In the vicinity of broad s -wave resonances, the resonant magnetic field is nearly independent on the transverse confining frequency ω_{\perp} of the waveguide. Close to p -wave and narrow s -wave resonances, the resonant magnetic field changes as ω_{\perp} varies.

1. Introduction

Ultracold atoms in deep optical lattices offer a highly controllable platform to study quantum systems in reduced dimensions [1]. The shape of the optical lattices can be engineered to realize different confining geometries [2] and the interatomic interaction can be tuned through magnetic Feshbach resonances [3]. As a result, ultracold atoms in quasi-one dimensional (quasi-1D) waveguides have been realized experimentally [4, 5, 6, 7] where the reduced dimensionality strongly affects the two-body bound and scattering states. In the waveguide, weakly bound molecular states exist for both positive and negative scattering length [4]. This is in contrast to free space where the weakly bound states exist only on the side of positive scattering length [3]. A prominent scattering property in waveguides is the confinement-induced resonance (CIR) [6, 8, 9] which is characterized by the divergence of the effective 1D interaction strength g_{1D} , and occurs at finite free space scattering length [10].

Theoretically, the s -wave [11, 12, 13, 14, 15] and p -wave [16] bound states in the

waveguide close to Feshbach resonances have been studied based on two-channel model. The resonant scattering properties in the waveguide have been investigated numerically by using a multichannel model [17]. In this work, the local frame transformation (LFT) [18, 19, 20] will be utilized to explore the two-body collisions in a harmonic waveguide. It has been shown [21, 22, 23, 24, 25, 26] that the LFT method is convenient to tackle high partial wave collisions in confined geometries and also to deal with couplings of different partial wave states due to the confinement. In previous studies using the LFT method [25, 26], the interatomic interaction is described by a single-channel potential. The short-range part of the potential is modified to mimick the variation of the phase shift close to Feshbach resonances. Within such a treatment, the influence of the resonance width is not accounted for. In the current work, the energy and magnetic field dependent phase shift induced by the multichannel interatomic interaction is incorporated into the LFT method, which allows one to calculate the two-body properties in the waveguide close to both broad and narrow resonances. The current LFT approach complements previous methods [12, 13, 14, 15] which have been dedicated to explore the confinement induced Feshbach molecules.

The weakly bound molecular state in a harmonic waveguide can be tuned by a magnetic field, and crosses the scattering threshold at the magnetic field where the effective 1D scattering length a_{1D} diverges. Accordingly, in a harmonic waveguide, the association of an unbound atom pair into a molecule occurs when the magnetic field is swept adiabatically across the pole of a_{1D} . For s -wave collisions, g_{1D} is inversely proportional to a_{1D} [27]. Hence, the molecular association occurs at $g_{1D} = 0$ other than at CIR. For p -wave collisions, g_{1D} is proportional to a_{1D} [27], and the molecular association occurs at the CIR. Here we assume that the two atoms possess the same mass. If the two masses are different, the center of mass and relative motions are coupled and the corresponding molecular formation process has been discussed in [28].

Our work is organized as follows. In Sec. II, we briefly review the local frame transformation approach, and show how the two-body properties in a waveguide are derived. Sec. III introduces the free space phase shift which is an ingredient of the LFT approach. In Sec. IV, the bound state and scattering properties in the waveguide are presented and discussed. Sec. V provides our conclusions.

2. The local frame transformation approach

We consider ultracold collisions of two identical atoms in a harmonic waveguide with cylindrical symmetry. An external magnetic field is applied to tune the interatomic interaction via free space Feshbach resonances [3]. The direction of the magnetic field is assumed to be parallel to the symmetry axis of the waveguide, namely the z axis. Due to the harmonic confinement, the center of mass motion and relative motion are separable. Due to this separation, all the relevant collisional physics is described by the

Hamiltonian of the relative degrees of freedom which reads as follows

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{1}{2}\mu\omega_\perp^2\rho^2 + V(\mathbf{r}), \quad (1)$$

where μ is the reduced mass, ω_\perp is the transverse confinement frequency, $r = \sqrt{z^2 + \rho^2}$ is the interatomic distance with z and ρ being the longitudinal and transverse components of the vector \mathbf{r} , respectively. $V(\mathbf{r})$ is the interatomic interaction.

The local frame transformation [25, 26] is employed to study the two-body properties in the waveguide. The key property of the LFT relies on the length scale separation between the confining potential and the interatomic interaction potential. More specifically, the length scale of the confinement is characterized by the harmonic length $a_\perp = \sqrt{\hbar/\mu\omega_\perp}$. The long-range interatomic interaction is considered to be the isotropic van der Waals interaction, and the anisotropic interaction such as magnetic dipole-dipole interaction is neglected. Then the length scale of the interatomic interaction is given by $\beta_6 = (2\mu C_6/\hbar^2)^{1/4}$, where C_6 is van der Waals coefficient [3]. The validity of the LFT is ensured by the length scale separation $a_\perp \gg \beta_6$. Due to this separation, two regions with different symmetries are identified. In the region $r \sim \beta_6$, the interatomic interaction is dominant and the confinement potential is negligible. The two-body collision is treated as free space scattering problem at energy E . The spherical symmetry in this region is exploited as the wavefunction is expanded in partial wave states. A single partial wave approximation is adopted which is justified due to the low energy collision considered here. All the scattering information in this region is contained in the free space K matrix $K^{3D} = \tan \eta$, where η is the free space scattering phase shift. To make sure that the phase shift η is well defined, the total energy E is considered to be positive $E > 0$. In the region $r \geq a_\perp$, the atoms feel the confinement potential whereas the interatomic interaction is negligible. The system possesses cylindrical symmetry, and the corresponding wavefunction is expressed as the product of the harmonic oscillator wavefunction in the transverse plane and a 1D plane wave in the longitudinal z -direction. The K matrix in this region is denoted as K^{1D} , which contains the bound and scattering properties in the presence of the waveguide. In the intermediate region $\beta_6 < r < a_\perp$, the local frame transformation matrix U [18, 19, 20] is used to connect the wavefunctions at short distances to that at large distances. The K^{1D} matrix in the waveguide is related to the free space K^{3D} matrix by $K^{1D} = UK^{3D}U^T$ [25]. The elements of the local transformation matrix read [25]

$$U_{l,n}^T = \frac{\sqrt{2}(-1)^{d_0}}{a_\perp} \sqrt{\frac{2l+1}{kq_n}} P_l\left(\frac{q_n}{k}\right), \quad (2)$$

where l is the partial wave quantum number, and n is the quantum number specifying the harmonic oscillator state in the transverse plane. d_0 equals $l/2$ when l is even, and is equal to $(l+1)/2$ when l is odd. $P_l(x)$ is the l th-order Legendre polynomial, and q_n is the longitudinal momentum given by $\frac{(\hbar q_n)^2}{2\mu} = E - \hbar\omega_\perp(2n+1)$. The quantum number related to the azimuthal symmetry in transverse waveguide directions is set to zero in the derivation of equation (2) [25].

For the energy region $0 < E < \hbar\omega_\perp$, i.e. the total energy E is smaller than the ground state energy of the transverse confinement, a possible bound state in the waveguide is determined by

$$\det(I - iK^{1D}) = 0. \quad (3)$$

For the energy region $\hbar\omega_\perp < E < 3\hbar\omega_\perp$ in which only the ground mode of the transverse confinement is occupied, the physical $K^{1D,\text{phys}}$ matrix is defined by properly eliminating all the energetically closed excited modes [25]

$$K^{1D,\text{phys}} = \frac{K^{3D}U_{l,n=0}^2}{1 - iK^{3D}\mathfrak{U}_{l,l}}, \quad (4)$$

where $\mathfrak{U}_{l,l} = \sum_{n=1}^{\infty} U_{l,n}^2$.

The collision in the waveguide can be mapped to a one-dimensional scattering problem [10]. For s -wave collisions in the waveguide, the effective 1D scattering length a_{1D}^s defined in [8] is related to $K^{1D,\text{phys}}$ by

$$a_{1D}^s = \lim_{q_0 \rightarrow 0} 1/(q_0 K^{1D,\text{phys}}). \quad (5)$$

The 1D interaction strength g_{1D}^s is expressed in terms of a_{1D}^s by $g_{1D}^s = -\hbar^2/(\mu a_{1D}^s)$ [10]. According to the definition in [29], the effective 1D scattering length for p -wave collisions in the waveguide is expressed as

$$a_{1D}^p = - \lim_{q_0 \rightarrow 0} K^{1D,\text{phys}}/q_0. \quad (6)$$

In contrast to the s -wave case in which g_{1D}^s is inversely proportional to a_{1D}^s , the effective 1D interaction strength for p -wave collisions is $g_{1D}^p = -\hbar^2 a_{1D}^p/\mu$ i.e. is proportional to a_{1D}^p [27].

3. Free space phase shift

As shown by equations (3)-(6), the free space K^{3D} matrix or equivalently the free space phase shift is needed to calculate the bound and scattering properties in the waveguide using the LFT method. In the ultracold regime, the energy and magnetic field dependent phase shift in the vicinity of a well-isolated s -wave Feshbach resonance is obtained via [30, 31]

$$-\frac{\tan \eta_s(E, B)}{k} = a_{\text{bg}} \left[1 + \frac{\Delta}{E/\delta\mu - (B - B_0)} \right], \quad (7)$$

where $\hbar k = \sqrt{2\mu E}$, B is the magnetic field strength, and a_{bg} is the background scattering length. B_0 and Δ are the resonance position and width, respectively. $\delta\mu$ is the difference between the magnetic moments of the incident scattering state and the bound state which produces the resonance. The two-body properties in the waveguide around two ^6Li s -wave Feshbach resonances are explored in the following. The values of B_0 , Δ , $\delta\mu$, a_{bg} and C_6 for the two s -wave resonances are listed in Table 1. The parameter $R^* = \frac{\hbar^2}{2\mu a_{\text{bg}} \delta\mu \Delta}$ [33] is also given in the table. $R^* \ll 1$ indicates a broad resonance, and the

Table 1. Parameters of the two ${}^6\text{Li}$ Feshbach resonances. The data are taken from [32].

	B_0 [G]	Δ [G]	$\delta\mu$ [μ_B]	a_{bg} [au]	C_6 [au]	R^* [au]
Resonance I	832	-262	1.87	-1593	1393.39	0.55
Resonance II	534.4	0.1	1.97	59	1393.39	3.69×10^4

closed-channel component in the wavefunction is small over the resonant width. $R^* \gg 1$ indicates a narrow resonance, and the closed-channel component in the wavefunction is dominant over a large fraction of the resonant width. Hereafter, the broad ${}^6\text{Li}$ Feshbach resonance at $B_0 = 832$ G will be referred to as resonance I, and the narrow ${}^6\text{Li}$ Feshbach resonance at $B_0 = 534.4$ G is denoted as resonance II.

The collision of two ${}^{40}\text{K}$ atoms in the waveguide close to a p -wave Feshbach resonance [34] is also studied in Section IV. The energy and magnetic field-dependent phase shift obtained in [34] is utilized in the calculation of the bound and scattering properties.

4. Bound and scattering properties in waveguides

4.1. s -wave case

Making use of the relation between $K^{1\text{D}}$ and $K^{3\text{D}}$, and after some algebra [25], the bound-state equation (3) for $l = 0$ can be written as

$$\frac{a_{\perp}}{a_{\text{eff}}(E, B)} = -\zeta\left(\frac{1}{2}, -\chi\right), \quad (8)$$

where $a_{\text{eff}}(E, B) = -\tan\eta_s(E, B)/k$ is the effective scattering length introduced in [35, 36], and η_s is the s -wave scattering phase shift. $-\chi = (\hbar\omega_{\perp} - E_b)/\hbar\omega_{\perp}$ is the scaled binding energy of the bound state with energy E_b , and $\zeta(s, p)$ is the Hurwitz zeta function. Combining equations (4) and (5), the 1D scattering length can be expressed as

$$a_{1\text{D}}^s(B) = -\frac{a_{\perp}}{2} \left[\frac{a_{\perp}}{a_{\text{eff}}(E = \hbar\omega_{\perp}, B)} + \zeta\left(\frac{1}{2}\right) \right], \quad (9)$$

and $\zeta(s)$ is the Riemann zeta function. The expressions for the bound state and $a_{1\text{D}}^s$ are the same with that from [10] except that the effective scattering length at the zero-point energy of the transverse confinement $E = \hbar\omega_{\perp}$ is used instead of the zero-energy scattering length.

The binding energy of two ${}^6\text{Li}$ atoms in a waveguide close to the broad resonance I is shown by the black solid line in the upper panel of figure 1. Besides the Feshbach dimer which induces the free space resonance, a confinement-induced background dimer emerges in the waveguide due to the negative free space background scattering length

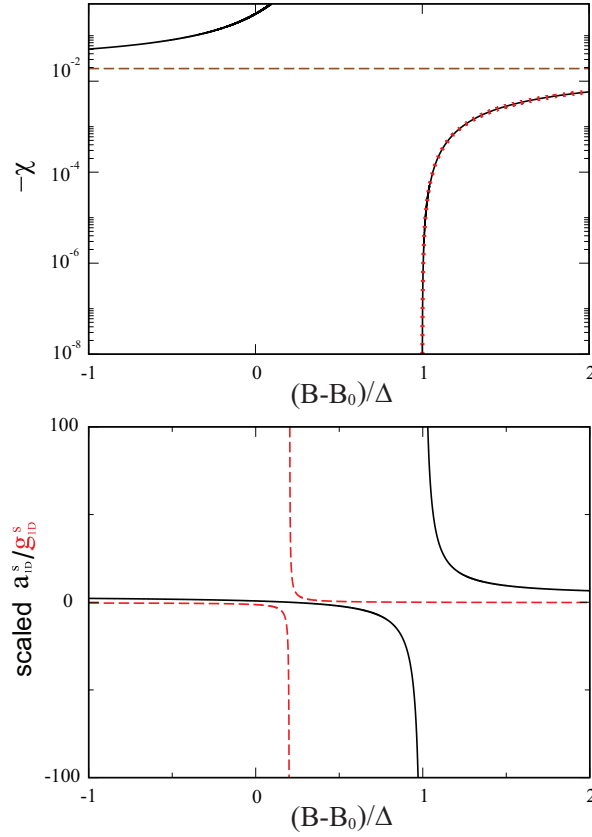


Figure 1. Bound and scattering properties for two ${}^6\text{Li}$ atoms in the waveguide in the vicinity of the s -wave Feshbach resonance at $B_0 = 832$ G. Upper panel: scaled binding energies calculated by equations (8) (black solid line) and (10) (red dotted line) are shown. The horizontal brown dashed line corresponds to $-\chi_{\text{bg}} = 0.0189$. Lower panel: 1D scattering length $a_{1\text{D}}^s$ scaled by a_{\perp} (black solid line) and 1D interaction strength $g_{1\text{D}}^s$ scaled by $\hbar^2/\mu a_{\perp}$ (red dashed line). The transverse trapping frequency ω_{\perp} is set to $2\pi \times 14$ kHz in the calculation, which is realized in [7]. The harmonic length a_{\perp} and van der Waals length β_6 are 9259 au and 63 au, respectively.

a_{bg} for resonance I [15]. Figure 1 shows the avoid crossing between the two bound states. The effective 1D scattering length $a_{1\text{D}}^s$ and the 1D interaction strength $g_{1\text{D}}^s$ are shown in the lower panel of figure 1 by the black solid and red dashed lines, respectively. One can see that the bound state crosses the threshold at the pole of $a_{1\text{D}}^s$ when $g_{1\text{D}}^s$ vanishes. This agrees with the observation in [17] that the bound state component in the scattering wavefunction is dramatically enhanced at the zero crossing of the free space scattering length a_{eff} , not at CIR. It is noted that the zero crossing of a_{eff} corresponds to infinite $a_{1\text{D}}^s$ according to equation (9). In the vicinity of the broad ${}^6\text{Li}$ resonance, the association of atom pairs into molecules in a harmonic waveguide is expected to happen by sweeping the magnetic field adiabatically across the region $B \sim B_0 + \Delta$.

For a weakly bound state $\chi \rightarrow 0^-$, $E = 2\hbar\omega_{\perp} \left(\frac{1}{2} + \chi\right) \sim \hbar\omega_{\perp}$, and the Hurwitz zeta function in equation (8) can be approximated by $\zeta\left(\frac{1}{2}, -\chi\right) \sim \frac{1}{\sqrt{-\chi}} + \zeta\left(\frac{1}{2}\right)$ [37]. Then the

binding energy of the weakly bound state can be expressed in terms of a_{1D}^s as

$$-\chi(B) = \frac{a_{\perp}^2}{4a_{1D}^s(B)^2}. \quad (10)$$

The binding energy calculated by equation (10) is shown in the upper panel of figure 1 by the red dotted line.

Substituting equation (7) for a_{eff} in equation (9), one can rewrite the expression as

$$a_{1D}^s = a_{\text{bg},1D} \left(1 - \frac{\Delta_{1D}}{B - B_{0,1D}} \right), \quad (11)$$

where $a_{\text{bg},1D} = -\frac{a_{\perp}^2}{2a_{\text{bg}}} \left(1 + \frac{a_{\text{bg}}}{a_{\perp}} \zeta\left(\frac{1}{2}\right) \right)$, $B_{0,1D} = B_0 + \Delta + \frac{\hbar\omega_{\perp}}{\delta\mu}$, and $\Delta_{1D} = -\Delta \left(1 + \frac{a_{\text{bg}}}{a_{\perp}} \zeta\left(\frac{1}{2}\right) \right)^{-1}$. Equations (10) and (11) are similar as their free space counterparts [3], and suggest that a Feshbach resonance in 1D [38] is realized effectively with two atoms colliding in the waveguide around a free space Feshbach resonance [11]. The resonant position of the Feshbach resonance in 1D is $B_{0,1D}$ which is shifted from the free space resonant position B_0 by $\Delta + \hbar\omega_{\perp}/\delta\mu$. It is to be noted that the term $\hbar\omega_{\perp}/\delta\mu$ originates from the energy-dependent term in equation (7). For broad resonances and close to the threshold, the energy-dependence of the phase shift is negligible [3], and hence the term $\hbar\omega_{\perp}/\delta\mu$ in the expression of $B_{0,1D}$ can be ignored. This statement is supported by the following quantitative analysis. Making use of the definition of R^* , the term $\hbar\omega_{\perp}/\delta\mu$ can be expressed as $\frac{a_{\text{bg}}R^*}{2a_{\perp}^2}\Delta$. Usually the background scattering length a_{bg} for two atoms colliding in free space is of the order of the van der Waals length β_6 , which is assumed to be far smaller than a_{\perp} . For broad resonances, R^* is small, and hence $\hbar\omega_{\perp}/\delta\mu$ is negligible compared to Δ . The 1D resonant position $B_{0,1D}$ is approximated to be $B_0 + \Delta$, which is independent on the transverse confinement frequency ω_{\perp} . This is verified in figure 1 for resonance I which shows that a_{1D}^s diverges at $(B - B_0)/\Delta \sim 1$. At $B = B_{0,1D} + \Delta_{1D}$, a_{1D}^s equals to zero, and g_{1D} tends to infinity. The width of the Feshbach resonance in 1D is Δ_{1D} , which has the same magnitude as the width Δ of the free space resonance, but a different sign. The background 1D scattering length is $a_{\text{bg},1D}$, which is of the order of a_{\perp} , and has the opposite sign of a_{bg} . For the ^6Li resonance considered in figure 1, the free space background scattering length a_{bg} is large and negative, i.e. we have $\Delta_{1D} = -0.8\Delta$, and $a_{\text{bg},1D}/a_{\perp} = 3.6$. The binding energy of the confinement induced bound state away from the resonant magnetic field $B_{0,1D}$ is estimated by substituting $a_{\text{bg},1D}$ into equation (10), which gives $-\chi_{\text{bg}} = \frac{a_{\perp}^2}{4a_{\text{bg},1D}^2} = 0.0189$. This value is depicted by the brown dashed line in the upper panel of figure 1.

An analogous investigation has been performed for two fermionic ^6Li atoms in a harmonic waveguide around the narrow resonance II, and the results are shown in figure 2. For resonance II, a_{bg} is positive, and $a_{\text{bg},1D}$ is negative. There is no confinement induced background dimer, and only the Feshbach dimer exists [15]. For narrow resonances, the phase shift is strongly energy-dependent [3], and the energy-dependent term in equation (7) cannot be neglected. Accordingly, the term $\hbar\omega_{\perp}/\delta\mu$ in the expression of $B_{0,1D}$ cannot be neglected. In figure 2, it is clearly shown that a_{1D}^s

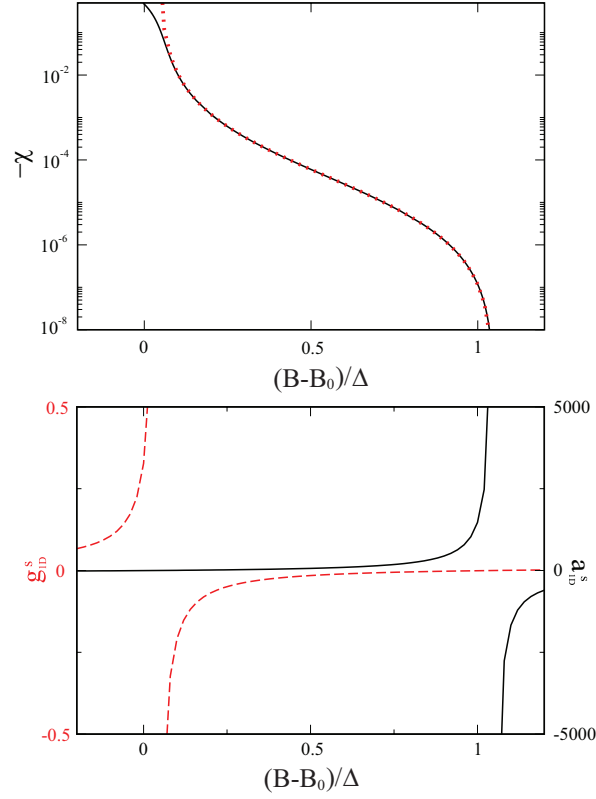


Figure 2. Bound and scattering properties in a waveguide in the vicinity of the ${}^6\text{Li}$ s -wave Feshbach resonance at $B_0 = 534.4$ G. Binding energies calculated by equations (8) (black solid line) and (10) (red dotted line) are shown in the upper panel. a_{1D}^s scaled by a_\perp (black solid line) and g_{1D}^s scaled by $\hbar^2/\mu a_\perp$ (red dashed line) are displayed in the lower panel. The confining frequency is the same as in figure 1.

diverges at a magnetic field strength different from $(B - B_0)/\Delta = 1$.

In free space, a weakly bound state exists only on the positive side of the free space scattering length close to resonance [3]. In the waveguide, it has been verified experimentally that the weakly bound state exists for both positive and negative free space scattering length [4]. Figures 1 and 2 show that, in terms of the effective one-dimensional scattering length a_{1D}^s , similar statements like those in free space can be made in the presence of the waveguide, i.e. the weakly bound state exists only on the positive side of a_{1D}^s .

4.2. p -wave case

Let us now focus on the p -wave interaction in the waveguide. Substituting K^{1D} by $UK^{3D}U^T$ and following the derivation in [21, 25], the bound-state equation (3) for $l = 1$ becomes

$$12\zeta\left(-\frac{1}{2}, -\chi\right) = \frac{a_\perp^3}{V_p(E, B)}, \quad (12)$$

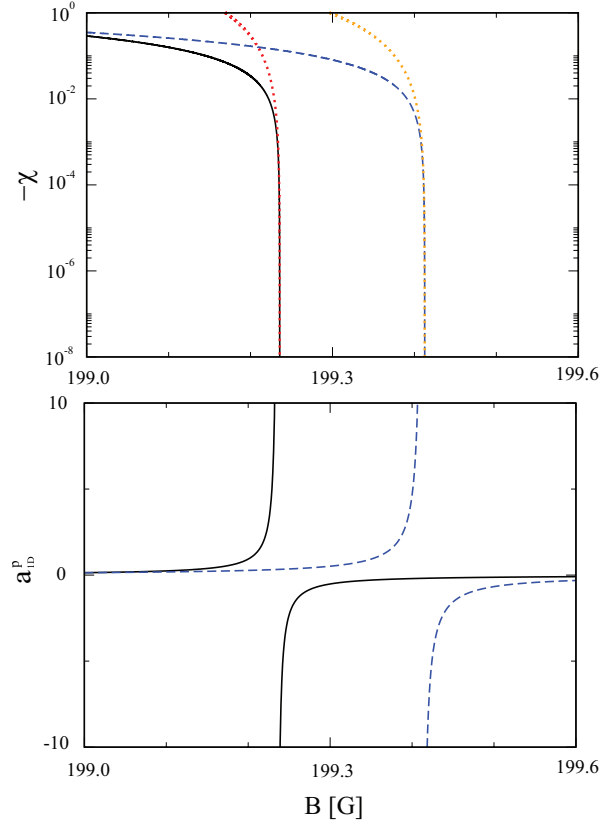


Figure 3. Binding energy (upper panel) and effective 1D scattering length (lower panel) in a harmonic waveguide close to the ^{40}K p -wave Feshbach resonance [34]. Results with confining frequency $\omega_{\perp} = 2\pi \times 69$ and 100 kHz are shown in black solid and blue dashed lines, respectively. The red and orange dotted lines in the upper panel are the binding energies calculated by equation (10). The 1D scattering length a_{1D}^p shown in the lower panel is scaled by a_{\perp} .

where $V_p(E, B) = -\tan \eta_p(E, B)/k^3$, and η_p is the p -wave scattering phase shift. According to equations (4) and (6), the effective 1D scattering length a_{1D}^p is

$$a_{1D}^p(B) = 6a_{\perp} \left[\frac{a_{\perp}^3}{V_p(E = \hbar\omega_{\perp}, B)} - 12\zeta\left(-\frac{1}{2}\right) \right]^{-1}, \quad (13)$$

which reproduces the corresponding expressions in [21, 29]. For a weakly bound state $\chi \rightarrow 0^-$, the Hurwitz zeta function in equation (12) can be approximated by $\zeta(-\frac{1}{2}, -\chi) \sim \sqrt{-\chi} + \zeta(-\frac{1}{2})$ [37]. The relation between the binding energy of the weakly bound state and 1D scattering length, equation (10), also applies for the p -wave case if a_{1D}^s is substituted by a_{1D}^p .

The binding energy and effective 1D scattering length for two ^{40}K atoms in a harmonic waveguide interacting in the vicinity of a p -wave Feshbach resonance [34] are shown in figure 3. Calculations are performed with two confining frequencies $\omega_{\perp} = 2\pi \times 69$ kHz (black solid line) and $2\pi \times 100$ kHz (blue dashed line). The binding energy determined via equation (10), which is valid for a weakly bound state, is

depicted in red and orange dotted lines in the upper panel. The results show that the confining frequency can be used to tune the bound state and scattering properties.

As in the s -wave case, the bound state in the waveguide crosses the scattering threshold at the magnetic field where a_{1D}^p diverges. For p -wave scattering, g_{1D}^p is proportional to a_{1D}^p , and a p -wave CIR occurs at the magnetic field strength where a_{1D}^p is divergent. Hence, the adiabatic molecular formation happens at the position of the CIR for p -wave interaction [5], in contrast to the s -wave case where the molecular formation occurs at $g_{1D}^s = 0$ other than at CIR.

5. Conclusion

The bound state and two-body collisions in a harmonic waveguide close to free space Feshbach resonances have been investigated by using the local frame transformation approach. As an extension to previous studies [21, 25, 26], the energy and magnetic field dependent free space phase shifts due to a realistic multichannel interatomic interaction are adopted in the LFT method. The LFT method, which relies on the length scale separation $\beta_6 \ll \omega_\perp$, complements the zero-range [12, 13, 14, 16] and finite-range [15] two-channel models which have been used to explore the two-body properties in the waveguide close to Feshbach resonances.

The position of the association of atoms into molecules in a harmonic waveguide during an adiabatic sweep of the magnetic field has been investigated, and its relation with the position of the confinement induced resonance, indicated by an infinite one-dimensional interaction strength g_{1D} , has been discussed. For both s and p wave interaction, the least bound state in the waveguide crosses the scattering threshold at the magnetic field $B_{0,1D}$ where the effective one-dimensional scattering length a_{1D} diverges. For the s -wave case, a_{1D} and g_{1D} are inversely proportional to each other, and the position of molecular formation differs from the position of the s -wave CIR. For p -wave interaction, a_{1D} is proportional to g_{1D} , and the position of the molecular formation coincides with the position of the p -wave CIR. Moreover, in the vicinity of broad s -wave Feshbach resonances, the magnetic field $B_{0,1D}$ is nearly independent on the transverse confining frequency ω_\perp , and is approximately $B_0 + \Delta$, where B_0 and Δ are the free space resonance position and width. Close to p -wave and narrow s -wave Feshbach resonances, the free space phase shift is strongly energy-dependent. The zero-point energy $\hbar\omega_\perp$ of the transverse confinement plays a role in determining the value of $B_{0,1D}$, and $B_{0,1D}$ changes as ω_\perp varies.

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